HW 5C: (HL) #2, 3, 5, 6beh, 7, 9, 10c, 12

5C: (SL) #4, 5, 7, 9beh, 10, 11c, 12c

2.) 5, 10, 20, 40, ... $\frac{10}{5} = \frac{20}{10} = \frac{40}{20} = \boxed{2}$

b.) $u_n = 5(2)^{n-1} \therefore u_{15} = 5(2)^{14} = \boxed{81,920}$

3.) a.) $12, -6, 3, \frac{-3}{2}, \ldots$ $\frac{-6}{12} = \frac{3}{-6} = \frac{\frac{-3}{2}}{3} = \boxed{-\frac{1}{2}}$

b.) $u_n = 12\left(\frac{-1}{2}\right)^{n-1} \therefore u_{13} = 12\left(\frac{-1}{2}\right)^{12} = 2^2 \cdot 3\left(\frac{1}{2^{12}}\right) = \frac{3}{2^{10}} = \boxed{\frac{3}{1,024}}$

5.) a.) $8, 4\sqrt{2}, 4, 2\sqrt{2}, \ldots$ $\frac{4\sqrt{2}}{8} = \frac{4}{4\sqrt{2}} = \frac{2\sqrt{2}}{4} = \frac{\sqrt{2}}{2}$

b.) $u_n = 2^{\frac{7}{2} - \frac{1}{2}n}$ - show...

$\hookrightarrow \frac{4}{4\sqrt{2}} \cdot \frac{\sqrt{2}}{\sqrt{2}} = \frac{\sqrt{2}}{2}$ ✓

start by plugging in u_1 & r to general term form

$u_n = 8\left(\frac{\sqrt{2}}{2}\right)^{n-1}$

$\Rightarrow$ rewrite 8 and $\frac{\sqrt{2}}{2}$ with common base 2

$\Rightarrow u_n = 2^3\left(2^{-\frac{1}{2}}\right)^{n-1}$

$\frac{\sqrt{2}}{2} = \frac{2^{\frac{1}{2}}}{2^1} = 2^{\frac{1}{2}-1} = 2^{-\frac{1}{2}}$

apply the power rule

$\Rightarrow u_n = 2^3 \cdot 2^{-\frac{1}{2}n + \frac{1}{2}}$

apply the product rule & combine like terms $\Rightarrow u_n = 2^{3 - \frac{1}{2}n + \frac{1}{2}}$

$\Rightarrow \boxed{u_n = 2^{\frac{7}{2} - \frac{1}{2}n}}$

$3 = \frac{6}{2} \therefore \frac{6}{2} + \frac{1}{2} = \frac{7}{2}$

6b.) $1000, 4k, k \Rightarrow \frac{4k}{1000} = \frac{k}{4k} \therefore 16k^2 = 1000k$

$\Rightarrow 16k^2 - 1000k = 0$

$\Rightarrow 8k(2k - 125) = 0$

$\therefore 8k = 0$ & $2k - 125 = 0$

$\boxed{\cancel{k=0}}$ $2k = 125$

1,000, 0, 0 is not geometric

$\boxed{k = 125/2}$

6e.) $k, 12, \frac{k}{9}$ $\therefore$ $\frac{12}{k} = \frac{\frac{k}{9}}{12}$ $\Rightarrow$ $\frac{12}{k} = \frac{k}{108}$ $\Rightarrow k^2 = 1296$

check: $k=36 \rightarrow 36, 12, 4$ $r = \frac{1}{3}$ ✓ $\therefore \boxed{k = \pm 36}$

$k=-36 \rightarrow -36, 12, -4$ $r = \frac{-1}{3}$ ✓

6h.) $k, k+8, 9k$ $\therefore$ $\frac{k+8}{k} = \frac{9k}{k+8}$ $\Rightarrow (k+8)(k+8) = 9k^2$

$k^2+8k+8k+64 = 9k^2$

$8k^2 - 16k - 64 = 0$ $\Leftarrow$ $k^2+16k+64 = 9k^2$

$8(k^2-2k-8) = 0$

-8, -4, 2, -2

$8(k-4)(k+2) = 0$ Check: $k=-2 \Rightarrow -2, 6, -18$ $r=-3$ ✓

$\therefore \boxed{k = -2, 4}$ $k=4 \Rightarrow 4, 12, 36$ $r = 3$ ✓

7.) $k-1, 6, 3k$ $\therefore$ $\frac{6}{k-1} = \frac{3k}{6}$ $\Rightarrow 3k(k-1) = 36$

-12, -4, 3, -1

$\therefore \boxed{k = -3, 4}$ $3k^2 - 3k - 36 = 0$

$3(k^2-k-12) = 0$

$3(k-4)(k+3) = 0$

b.) When $k=-3 \rightarrow -4, 6, -9$ & $r = \frac{-3}{2}$ $\therefore -9 \cdot \frac{-3}{2} = \boxed{\frac{27}{2}}$

When $k=4 \rightarrow 3, 6, 12$ & $r=2$ $\therefore 12 \cdot 2 = \boxed{24}$

9.) $u_3 = 80$ $u_6 = 270$ $\therefore$ $80 = u_1 \cdot r^{3-1} \rightarrow u_1 r^2 = 80$

$270 = u_1 \cdot r^{6-1} \rightarrow u_1 r^5 = 270$

$\Rightarrow \frac{\cancel{u_1} r^5}{\cancel{u_1} r^2} = \frac{270}{80}$ $\Rightarrow \sqrt[3]{r^3} = \sqrt[3]{\frac{27}{8}}$ $\boxed{r = \frac{3}{2}}$ $u_1\left(\frac{3}{2}\right)^2 = 80$

$\therefore u_1 = \boxed{\frac{320}{9}}$ $u_1\left(\frac{9}{4}\right) = 80 \Rightarrow u_1 = 80\left(\frac{4}{9}\right)$

$\therefore \boxed{u_n = \frac{320}{9}\left(\frac{3}{2}\right)^{n-1}}$

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10c.) $12, 6, 3, 1.5, \ldots$ less than 0.0001

$r = \frac{1}{2}$ $\quad \therefore \; 12\left(\frac{1}{2}\right)^{n-1} < 0.0001$

$\therefore \boxed{n = 18}$ $\quad \frac{1}{2}^{n-1} < \left(\frac{0.0001}{12}\right) \rightarrow \log_{\frac{1}{2}}\left(\frac{0.0001}{12}\right) = n-1$

Check: when $n=17 \rightarrow 12\left(\frac{1}{2}\right)^{16} = 1.831E^{-4}$ $\quad \rightarrow 16.9 = n-1$

$= 0.0001831$ $\quad \rightarrow n = 17.9$

when $n = 18 \rightarrow 12\left(\frac{1}{2}\right)^{17} = 9.155E^{-5}$

$= 0.00009155$ ✓

12.) u_1 & u_2 – same for both sequences

u_3 – ratio $2:1$

a.) arithmetic: $u_1, \; u_1 + d, \; u_1 + 2d$

geometric: $u_1, \; u_1 \cdot r, \; u_1 \cdot r^2$

set expressions for u_2 equal to eachother

$\therefore u_1 + d = u_1 \cdot r$

$\rightarrow d = u_1 \cdot r - u_1$

$\boxed{d = u_1(r-1)}$

u_3 has ratio $2:1 \therefore u_1 r^2 = 2(u_1 + 2d)$

$\rightarrow u_1 r^2 = 2u_1 + 4d$ substitute $u_1(r-1)$ for d

$\rightarrow u_1 r^2 = 2u_1 + 4u_1(r-1)$

$\rightarrow r^2 = 2 + 4(r-1) \rightarrow r^2 = 2 + 4r - 4$

$\rightarrow r^2 = 4r - 2$ complete the square to solve...

$r^2 - 4r + \underline{(-2)^2} = -2 + \underline{4}$

$\left(\frac{-4}{2}\right)^2 = (-2)^2$

$(r-2)^2 = 2$

$r - 2 = \pm\sqrt{2}$

$\boxed{r = 2 \pm \sqrt{2}}$

b.) when $r = 2 + \sqrt{2}$, $d = u_1(2 + \sqrt{2} - 1)$

arithmetic $\quad \rightarrow d = u_1(1 + \sqrt{2})$

$\therefore u_4 = u_1 + 3u_1(1 + \sqrt{2})$

$= u_1(1 + 3(1+\sqrt{2})) \rightarrow u_1(1 + 3 + 3\sqrt{2})$

$\rightarrow u_4 = u_1(4 + 3\sqrt{2})$

geometric: $u_4 = u_1(2+\sqrt{2})^3 \Rightarrow u_1(2+\sqrt{2})(2+\sqrt{2})^2$

$\Rightarrow u_1(2+\sqrt{2})(6+4\sqrt{2})$

$\Rightarrow u_1(20 + 14\sqrt{2})$ Go to the top of 12

$\Rightarrow u_4 = 2u_1(10 + 7\sqrt{2})$

$(2+\sqrt{2})(2+\sqrt{2})$

$4 + \underline{2\sqrt{2} + 2\sqrt{2}} + 2 = 6 + 4\sqrt{2}$

$= (6 + 4\sqrt{2})(2 + \sqrt{2})$

$12 + 6\sqrt{2} + 8\sqrt{2} + 8 = 20 + 14\sqrt{2}$

$\therefore \frac{2\cancel{u_1}(10+7\sqrt{2})}{\cancel{u_1}(4+3\sqrt{2})}$

$= \frac{2(10+7\sqrt{2})(4-3\sqrt{2})}{(4+3\sqrt{2})(4-3\sqrt{2})}$

$= \frac{2(40 - 30\sqrt{2} + 28\sqrt{2} - 42)}{16 - 18}$

$= \frac{2(-2-2\sqrt{2})}{-2} = 2 + 2\sqrt{2}$ $\therefore$ the ratio of u_4's is $\boxed{2 + 2\sqrt{2} : 1}$