HW (HL) 5H: 1c, 2c, 3ab, 8, 9, 12 5I: 2ab, 4a, 6,
(SL) 5H: 2c, 3c, 5ab, 9, 10, 11 5I: 2ab, 4a, 6

5H: 1c.) $12+6+3+1.5+\ldots$ $\therefore S_n = \frac{u_1(1-r^n)}{1-r} \Rightarrow \frac{12(1-(\frac{1}{2})^{10})}{1-\frac{1}{2}}$

$r=\frac{1}{2}$ $u_1=12$ $n=10$

$\Rightarrow \frac{12(1-(\frac{1}{2})^{10})}{\frac{1}{2}} \Rightarrow 24(1-(\frac{1}{2})^{10})$

$\approx \boxed{24.0}$

2c.) $0.9+0.09+0.009+0.0009+\ldots$ $S_n = \frac{9/10(1-(\frac{1}{10})^n)}{(1-1/10)}$

$r=\frac{1}{10}$ $u_1=\frac{9}{10}$ $n=n$

$S_n = \frac{\cancel{9/10}(1-(1/10)^n)}{\cancel{9/10}}$

$\boxed{\Rightarrow S_n = 1-(1/10)^n}$

3a.) $\sum_{k=1}^{10} 3\times 2^{k-1}$ $u_1=3$ $r=2$ $n=10$ $S_n = \frac{3((2)^{10}-1)}{(2-1)}$

$\Rightarrow S_n = \frac{3((2)^{10}-1)}{1} = 3((2)^{10}-1) = \boxed{3069}$

3b.) $\sum_{k=1}^{12} (\frac{1}{2})^{k-2}$ pay attention $u_1=2$ $r=\frac{1}{2}$ $n=12$ $S_n = \frac{1(1-(\frac{1}{2})^{12})}{1-1/2}$

$\Rightarrow S_n = \frac{2(1-(1/2)^{12})}{1/2} \Rightarrow S_n = 4(1-(1/2)^{12}) \Rightarrow \approx \boxed{4.00}$

8.) $S_n = \frac{1}{2}+\frac{1}{4}+\frac{1}{8}+\frac{1}{16}+\ldots+\frac{1}{2^n}$

a.) $S_1 = \boxed{\frac{1}{2}}$ $S_2 = \frac{1}{2}+\frac{1}{4} = \frac{2}{4}+\frac{1}{4} = \boxed{\frac{3}{4}}$ $S_3 = \frac{1}{2}+\frac{1}{4}+\frac{1}{8} = \frac{4}{8}+\frac{2}{8}+\frac{1}{8} = \boxed{\frac{7}{8}}$

$S_4 = \frac{1}{2}+\frac{1}{4}+\frac{1}{8}+\frac{1}{16} = \frac{8}{16}+\frac{4}{16}+\frac{2}{16}+\frac{1}{16} = \boxed{\frac{15}{16}}$

$S_5 = \frac{1}{2}+\frac{1}{4}+\frac{1}{8}+\frac{1}{16}+\frac{1}{32} = \frac{16}{32}+\frac{8}{32}+\frac{4}{32}+\frac{2}{32}+\frac{1}{32} = \boxed{\frac{31}{32}}$

b.) $\boxed{S_n = \frac{2^n-1}{2^n}}$

c.) $S_n = \frac{1/2(1-(1/2)^n)}{(1-1/2)}$

$S_n = \frac{\cancel{1/2}(1-(1/2)^n)}{\cancel{1/2}}$

$\boxed{S_n = 1-(1/2)^n}$

d.) as n gets very large, $(1/2)^n$ gets very small, thus, S_n gets closer and closer to 1.

e.) as $n \to \infty$, the sum of the fractions approaches the area of a 1x1 square unit.

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9.) $u_2 = 6 \quad S_3 = -14 \quad$ Find u_4

$6 = u_1 r \qquad u_1 + 6 + 6r = -14$

$\therefore r = \frac{6}{u_1} \Rightarrow u_1 + 6 + \frac{36}{u_1} = -14 \Rightarrow \left(u_1 + \frac{36}{u_1} + 20 = 0\right)(u_1)$

$\Rightarrow u_1^2 + 20u_1 + 36 = 0$

$\Rightarrow (u_1+2)(u_1+18) = 0$

$\therefore u_1 = -2$ or $u_1 = -18$

36, 2, 18, 20

If $u_1 = -2$, $(-2) + 6 + (-18) = -14$ ✓ $r = -3$ ∴ $\boxed{u_4 = 54}$

If $u_1 = -18$, $(-18) + 6 + (-2) = -14$ ✓ $r = -1/3$ ∴ $\boxed{u_4 = 2/3}$

12.) $u_1 = 6 \quad r = 1.5 \quad S_n = 79.125 \quad$ Find n

$\therefore 79.125 = \frac{6(1.5^n - 1)}{1.5 - 1} \Rightarrow 79.125 = \frac{6(1.5^n - 1)}{0.5}$

$\Rightarrow \frac{79.125}{12} = \frac{\cancel{12}(1.5^n - 1)}{\cancel{12}} \Rightarrow 6.59375 = 1.5^n - 1$

$+1 \qquad +1$

$\Rightarrow 7.59375 = 1.5^n \Rightarrow \log_{1.5}(7.59375) = n$

∴ $\boxed{n = 5}$

5I: 2a.) $0.\overline{4} = 0.4 + 0.04 + 0.004 + \ldots \quad u_1 = \frac{4}{10} \quad r = \frac{1}{10} \quad n = \infty$

$S_n = \frac{u_1}{1-r} = \frac{4/10}{1 - 1/10} = \frac{4/10}{9/10} = \frac{4}{10} \cdot \frac{10}{9} = \boxed{\frac{4}{9}}$

2b.) $0.\overline{16} = 0.16 + 0.0016 + 0.000016 + \ldots \quad u_1 = \frac{16}{100} \quad r = \frac{1}{100} \quad n = \infty$

$S_n = \frac{16/100}{1 - 1/100} = \frac{16/100}{99/100} = \frac{16}{\cancel{100}} \cdot \frac{\cancel{100}}{99} = \boxed{\frac{16}{99}}$

4a.) $18 + 12 + 8 + \frac{16}{3} + \ldots \quad u_1 = 18 \quad r = \frac{2}{3} \quad \therefore S_n = \frac{18}{1 - 2/3} = \frac{18}{1/3}$

$= 18 \cdot 3 = \boxed{54}$

6.) $S_3 = 19 \quad S_n = 27 \rightarrow 27 = \frac{u_1}{1-r} \rightarrow$

$27(1-r) = u_1$

$27(1 - 2/3) = u_1 \quad \therefore \boxed{u_1 = 9}$

$27(1/3) = u_1$

$27(1-r) + 27(1-r)r + 27(1-r)r^2 = 19$

$27 - \cancel{27r} + \cancel{27r} - \cancel{27r^2} + \cancel{27r^2} - 27r^3 = 19 \rightarrow 27 - 27r^3 = 19 \rightarrow \frac{27(1-r^3)}{27} = \frac{19}{27}$

$1 - r^3 = 19/27 \rightarrow -r^3 = -8/27 \rightarrow \sqrt[3]{r^3} = \sqrt[3]{8/27} \therefore \boxed{r = 2/3}$

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