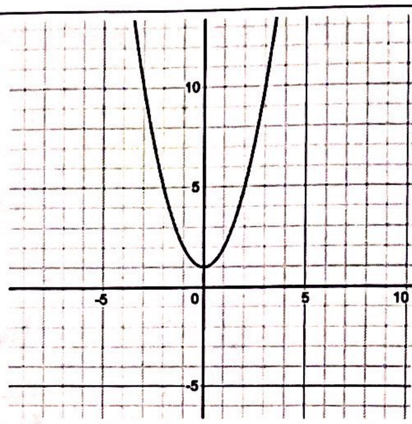
Name: Key

Algebra II

Period: ________

Imaginary/Complex Numbers

- Equations such as $x^2 + 1 = 0$, shown to the right, have "no real solutions" since their graphs don't cross the x-axis. So, mathematicians defined the imaginary numbers to represent their solutions.
- The **imaginary unit**, i, is defined as $\sqrt{-1}$. This is useful when working with square roots of negative numbers.
- A **complex number** is written in the form $a+bi$, where a is the real number and bi is the imaginary part.



Simplifying Square Roots

Example 1: $\sqrt{-4}$	Example 2: $\sqrt{-10}$	Example 3: $\sqrt{-20}$
$\sqrt{-1 \cdot 4}$	$\sqrt{-1 \cdot 10}$	$\sqrt{-1 \cdot 4 \cdot 5}$
$\sqrt{-1} \cdot \sqrt{4}$	$\sqrt{-1} \cdot \sqrt{10}$	$\sqrt{-1} \cdot \sqrt{4} \cdot \sqrt{5}$
$i \cdot 2 = \mathbf{2i}$	$i \cdot \sqrt{10} = \mathbf{i\sqrt{10}}$	$i \cdot 2 \cdot \sqrt{5} = \mathbf{2i\sqrt{5}}$

Directions: Solve each equation using the square root method.

1.) $x^2 + 9 = 0$	2.) $x^2 + 18 = 0$	3.) $2x^2 + 50 = 0$	4.) $3x^2 + 24 = 0$
$-9 \quad -9$ $\sqrt{x^2} = \sqrt{-9}$ $\boxed{x = \pm 3i}$	$-18 \quad -18$ $\sqrt{x^2} = \sqrt{-18}$ $\boxed{x = \pm 3i\sqrt{2}}$	$-50 \quad -50$ $\frac{2x^2}{2} = \frac{-50}{2}$ $\sqrt{x^2} = \sqrt{-25}$ $\boxed{x = \pm 5i}$	$-24 \quad -24$ $\frac{3x^2}{3} = \frac{-24}{3}$ $\sqrt{x^2} = \sqrt{-8}$ $\boxed{x = \pm 2i\sqrt{2}}$

Powers of i

$i^1 = i$ $\quad$ $i^2 = -1$ $\quad$ $i^3 = -i$ $\quad$ $i^4 = 1$

Directions: Simplify the following expressions.

5.) $3i + 4i$	6.) $2i \times 7i$	7.) $8i^3$	8.) $3i \times 2i \times i^2$
$\boxed{7i}$	$14i^2 = 14(-1)$ $= \boxed{-14}$	$8(-i)$ $= \boxed{-8i}$	$= 6i^4$ $= 6(1)$ $= \boxed{6}$
9.) $7i - i$	**10.) $5i^2 + 10i$**	**11.) $12i - 3i + 2i$**	**12.) $8i^2 \times 3i^2$**
$\boxed{6i}$	$5(-1) + 10i$ $= \boxed{-5 + 10i}$	$\boxed{11i}$	$24i^4$ $= 24(1)$ $= \boxed{24}$

Name: Key Algebra II Period: ________

Operations with Complex Numbers

Adding and Subtracting Complex Numbers

Directions: Simplify the expression below. Final answers must be in $a + bi$ form.

1.) $(-11 + 3i) + (9 + 2i)$

$\boxed{-2+5i}$

2.) $(4 + i) + (7 - 5i)$

$\boxed{11-4i}$

3.) $(7 - 2i) - (2 + 6i)$

$7-2i-2-6i$

$\boxed{5-8i}$

4.) $6i - (14 - i) + (5 - 3i)$

$6i-14+i+5-3i$

$\boxed{-9+4i}$

Multiplying Complex Numbers

Directions: Simplify the expression below. Final answers must be in $a + bi$ form.

5.) $2i(8 - 3i)$

$16i-6i^2$

$16i-6(-1)$

$16i+6 \Rightarrow \boxed{6+16i}$

6.) $-i(-2 + 10i)$

$2i-10i^2$

$2i-10(-1)$

$2i+10 \Rightarrow \boxed{10+2i}$

7.) $(7 + i)(4 - i)$

$28-7i+4i-i^2$

$28-3i+1$

$\boxed{29-3i}$

8.) $(2 - 4i)(-5 - 3i)$

$-10-6i+20i+12i^2$

$-10+14i-12$

$\boxed{-22+14i}$

Recall

$i^2=-1$

9.) $(6 - 2i)^2$

$(6-2i)(6-2i)$

$36-12i-12i+4i^2$

$36-24i-4$

$\boxed{32-24i}$

10.) $(5 + 4i)^2$

$(5+4i)(5+4i)$

$25+20i+20i+16i^2$

$25+40i-16$

$\boxed{9+40i}$

Complex Conjugates

Two complex numbers in the form $\boldsymbol{a + bi}$ and $\boldsymbol{a - bi}$ are called complex conjugates. The product of two conjugates is always a real number.

11.) $(8 + i)(8 - i)$

$64 \cancel{-8i} \cancel{+8i} - i^2$

$64+1$

$\boxed{65}$

12.) $(5 - 4i)(5 + 4i)$

$25 \cancel{+20i} \cancel{-20i} - 16i^2$

$25+16$

$\boxed{41}$