Name: Key

Date: ____________

Factoring
Common Core Algebra II

In the study of algebra there are certain skills that are called "gateway skills" because without them a student simply cannot enter into many more complex and interesting problems. Perhaps the most important gateway skill is that of **factoring**. The definition of factor, in two forms, is given below.

Factor – Two Important Meanings

(1) **Factor** (*verb*) – To rewrite a quantity as an equivalent product.
(2) **Factor** (*noun*) – Any individual component of a product.

You should be familiar with factoring integers as well as algebraic expressions from earlier courses. We will review some of the basic concepts and techniques of factoring in this lesson.

***Exercise* #1:** Factor each of the following integers completely. In other words, write them as the product of only prime numbers (called prime factorization).

(a) 12 $= 2\cdot 2\cdot 3$ (4, 3; 2, 2)

(b) 30 $= 2\cdot 3\cdot 5$ (6, 5; 2, 3)

(c) 16 $= 2\cdot 2\cdot 2\cdot 2$ (8, 2; 2, 4; 2, 2)

(d) 36 $= 2\cdot 2\cdot 3\cdot 3$ (6, 6; 2, 3; 2, 3)

Always keep in mind that when we **factor** (***verb***) a quantity, we are simply rewriting it in an different form that is completely equal to the original quantity. It might look different, but $2\cdot 3$ is still the number 6.

***Exercise* #2:** Rewrite each of the following binomials as a product of an integer with a different binomial.

(a) $5x+10$ $5(x+2)$

(b) $2x-6$ $2(x-3)$

(c) $6x+15$ $3(2x+5)$

(d) $6-14x$ $2(3-7x)$

The above type of factoring is often referred to as "factoring out" the greatest common factor (gcf). This greatest common factor can be comprised of numbers, variables, or both.

***Exercise* #3:** Write each of the following binomials as the product of the binomial's gcf and another binomial.

(a) $3x^2+6x$ $3x(x+2)$

(b) $20x-5x^2$ $5x(4-x)$

(c) $10x^2+25x$ $5x(2x+5)$

(d) $30x^2-20$ $10(3x^2-2)$

***Exercise* #4:** Rewritten in factored form $20x^2-36x$ is equivalent to

$4x(5x-9)$

(1) $2x(10x-15)$

(2) $4x(5x-9)$

(3) $5x(4x+7)$

(4) $9x(x-4)$

2.

Trinomials can also sometimes be factored into the product of a gcf and another trinomial.

***Exercise* #5:** Rewrite each of the following trinomials as the product of its gcf and another trinomial.

(a) $2x^2+8x+10$

$2(x^2+4x+5)$

(b) $10x^2-20x+5$

$5(2x^2-4x+1)$

(c) $8x^3-12x^2+20x$

$\cancel{4(2x^3-3x+5)}$

$4x(2x^2-3x+5)$

(d) $6x^3+15x^2-21x$

$3x(2x^2+5x-7)$

Another type of factoring that you should be familiar with stems from work on multiplying conjugates. Recall the conjugate multiplication pattern shown to the right. This can be "reversed" in order to factor binomials that have the form of the **difference of perfect squares**.

CONJUGATE MULTIPLICATION PATTERN
$(x-a)(x+a)=x^2-a^2$

***Exercise* #6:** Write each of the following binomials as the product of a conjugate pair.

(a) x^2-9

$(x+3)(x-3)$

(b) $4-x^2$

$(2-x)(2+x)$

(c) $4x^2-25$

$(2x+5)(2x-5)$

(d) $16-81x^2$

$(4-9x)(4+9x)$

***Exercise* #7:** Write each of the following binomials as the product of a conjugate pair.

(a) $x^2-\frac{1}{4}$

$(x+\frac{1}{2})(x-\frac{1}{2})$

(b) $25-\frac{x^2}{9}$

$(5+\frac{x}{3})(5-\frac{x}{3})$

(c) $\frac{4}{81}x^2-\frac{49}{9}$

$(\frac{2}{9}x-\frac{7}{3})(\frac{2x}{9}+\frac{7}{3})$

(d) $36x^2-49y^2$

$(6x+7y)(6x-7y)$

Factoring an expression until it cannot be factored anymore is known as **complete factoring**. Complete factoring is an important skill to master in order to solve a variety of problems. In general, when completely factoring an expression, the **first** type of factoring always to consider is that of **factoring out the gcf.**

***Exercise* #8:** Using a combination of gcf and difference of perfect squares factoring, write each of the following in its completely factored form.

(a) $5x^2-20$

$5(x^2-4)$

$5(x+2)(x-2)$

(b) $28x^2-7$

$7(4x^2-1)$

$7(2x^2+1)(2x^2-1)$

(c) $40-250x^2$

$10(4-25x^2)$

$10(2+5x^2)(2-5x^2)$

(d) $3x^3-48x$

$3x(x^2-16)$

$3x(x+4)(x-4)$