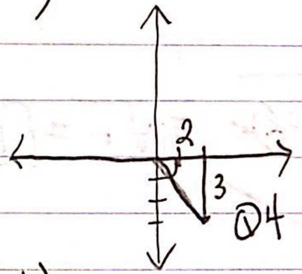
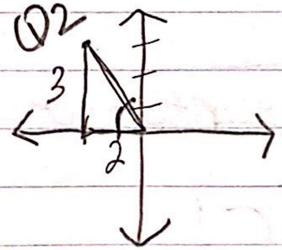
HW 11F # 3cd, 5 adgj, 6, 10

3c.) $2-3i$ $\quad |w| = \sqrt{(2)^2+(-3)^2} = \sqrt{4+9} = \boxed{\sqrt{13}}$



$\arg w = -\tan^{-1}\left(\frac{3}{2}\right) = \boxed{-0.983 \text{ radians}}$

3d.) $-2+3i$ $\quad |w| = \sqrt{(-2)^2+(3)^2} = \sqrt{4+9} = \boxed{\sqrt{13}}$



$\pi - \tan^{-1}\left(\frac{3}{2}\right) \approx \boxed{2.16 \text{ radians}}$

5.) If $z = 2+i$, $w = -1+3i$

a.) $|z| = \sqrt{(2)^2+(1)^2} = \sqrt{4+1} = \boxed{\sqrt{5}}$

d.) $zz^* = |z|^2 \therefore (\sqrt{5})^2 = \boxed{5}$

proof: $(2+i)(2-i) = 4 - i^2 = 4+1 = 5$ ✓

g.) $\left|\frac{z}{w}\right| = \frac{|z|}{|w|} = \frac{\sqrt{5}}{\sqrt{(-1)^2+(3)^2}} = \frac{\sqrt{5}}{\sqrt{1+9}} = \frac{\sqrt{5}}{\sqrt{10}} = \sqrt{\frac{5}{10}} = \sqrt{\frac{1}{2}} = \boxed{\frac{1}{\sqrt{2}}}$ ✓

proof: $\frac{z}{w} = \frac{(2+i)}{(-1+3i)}\frac{(-1-3i)}{(-1-3i)} \Rightarrow \frac{-2-6i-i-3i^2}{1-9i^2} = \frac{-2+3-7i}{1+9}$

$= \frac{1-7i}{10} \Rightarrow \left|\frac{1}{10} - \frac{7}{10}i\right| = \sqrt{\left(\frac{1}{10}\right)^2+\left(\frac{7}{10}\right)^2} = \sqrt{\frac{1}{100}+\frac{49}{100}}$

$\Rightarrow \sqrt{\frac{50}{100}} = \sqrt{\frac{1}{2}} = \boxed{\frac{1}{\sqrt{2}}}$ ✓

j.) $|z|^2 = zz^* = \boxed{5}$ (same as d.)

6.) Find $|z|$ given:

a.) $z = \cos\theta + i\sin\theta$ $|z| = \sqrt{\cos^2\theta+\sin^2\theta} = \sqrt{1} = \boxed{1}$

by property, $\cos^2\theta + \sin^2\theta = 1 \therefore$

b.) $z = r(\cos\theta + i\sin\theta)$, $r \in \mathbb{R}$

$\therefore z = r\cos\theta + ri\sin\theta$ $|z| = \sqrt{r^2\cos^2\theta + r^2\sin^2\theta}$

$\Rightarrow \sqrt{r^2(\cos^2\theta+\sin^2\theta)} = \sqrt{r^2(1)} = \sqrt{r^2} = \boxed{r}$

10.) $|z| = 3$, $\arg z = \frac{2\pi}{5}$

a.) $|w| = 3$ (because based on the graph w and z are the same length)

oops, out of order :)

c.) $|z+w| = \sqrt{(3)^2+(3)^2} = \sqrt{9+9} = \sqrt{18} = \boxed{3\sqrt{2}}$
(based on Pythagorean theorem)

b.) $\arg w = \arg z + \frac{\pi}{2}$ based on the diagram because $90° = \frac{\pi}{2}$

$\therefore \frac{2\pi}{5} + \frac{\pi}{2} = \frac{4\pi}{10} + \frac{5\pi}{10} = \boxed{\frac{9\pi}{10}}$

d.) $\arg(z+w) = \frac{2\pi}{5} + \tan^{-1}\left(\frac{3}{3}\right) \Rightarrow \frac{2\pi}{5} + \tan^{-1}(1)$

$\Rightarrow \frac{2\pi}{5} + \frac{\pi}{4}$ (based on unit circle)

$\Rightarrow \frac{8\pi}{20} + \frac{5\pi}{20} = \boxed{\frac{13\pi}{20}}$