HW 11G: 1b, 2, 3cd, 4a

1b.) $A(-4, 7)$ and $B(1, -3)$

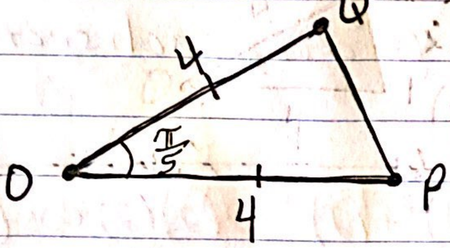
i. $AB = |(-4+7i)-(1-3i)| = |-5+10i| = \sqrt{(-5)^2+(10)^2}$

$= \sqrt{25+100} = \sqrt{125} = \sqrt{25 \cdot 5} = \boxed{5\sqrt{5} \text{ units}}$

ii. midpoint $AB = \dfrac{-4+7i+1-3i}{2} = \dfrac{-3+4i}{2} = \boxed{\left(\frac{-3}{2}, 2\right)}$

2.) $|z_1| = |z_2| = 4$ (radius of the circle)

a.) $|z_1 - z_2|$ = the distance between points P & Q.



using the cosine rule:

$PQ^2 = 4^2 + 4^2 - 2(4)(4)\cos\left(\frac{\pi}{5}\right)$

$\therefore PQ = \sqrt{16+16-32\cos\left(\frac{\pi}{5}\right)}$

$\Rightarrow \boxed{PQ = \sqrt{32-32\cos\left(\frac{\pi}{5}\right)}}$

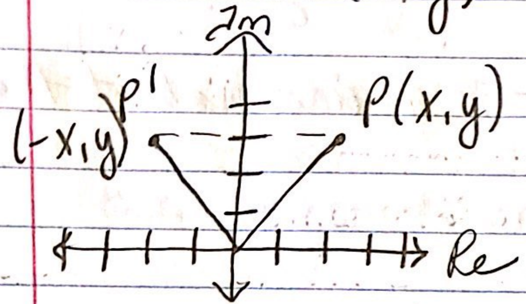
b.) perimeter $= 4+4+\sqrt{32-32\cos\left(\frac{\pi}{5}\right)} \Rightarrow \boxed{8+\sqrt{32-32\cos\left(\frac{\pi}{5}\right)} \text{ units}}$

Area $= \frac{1}{2}(4)(4)\sin\left(\frac{\pi}{5}\right) \rightarrow = \boxed{8\sin\left(\frac{\pi}{5}\right)}$ units^2

3c.) z to $-z^*$ If $z = x + yi$, then $z^* = x - yi$ and

$-z^* = -x + yi$

$\therefore P(x,y) \rightarrow P'(-x,y)$

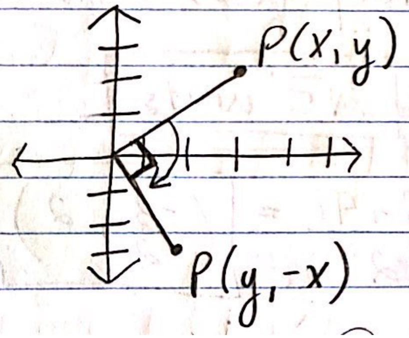


reflection about Im axis

3d.) z to $-iz$ If $z = x + yi$, then $-iz = -xi - yi^2$

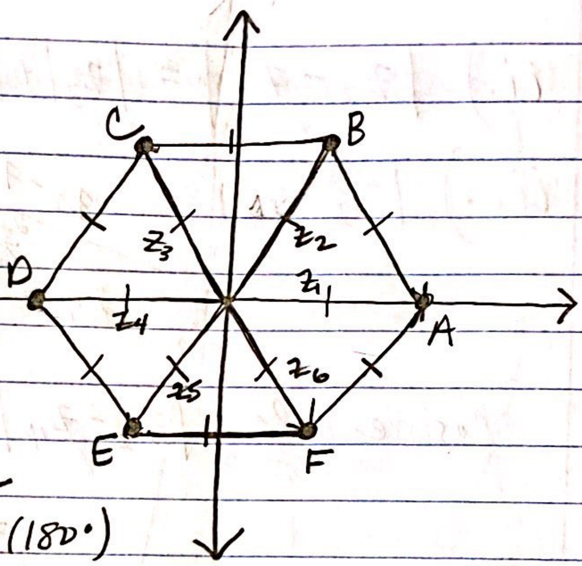
$-iz = y - xi$

$\therefore P(x,y) \rightarrow P'(y,-x)$



clockwise rotation of $\frac{\pi}{2}$ about the origin

4.) In a regular hexagon, all sides are the same length which means it forms 6 equalitareal triangles of the same dimensions.



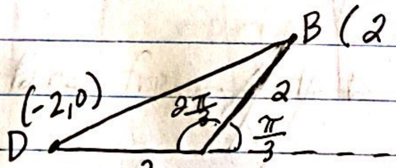
Also, that means that $\arg z_2$ is $\frac{\pi}{3}$ since all interior angles of the equilateral triangle are the same and add to be π. (180°)

Therefore,

i.) $|z_1 - z_2|$ (the distance between B & A) is the same as $|z_1|$ which is $\boxed{2}$ (they give us $|z_1| = 2$)

ii.) $|z_3 - z_6|$ (the distance from C to F) is $2 + 2 = \boxed{4}$

iii.) $|z_2 - z_4| = BD$



$B\left(2\cos\frac{\pi}{3}, 2\sin\frac{\pi}{3}\right) \to \left(2\left(\frac{1}{2}\right), 2\left(\frac{\sqrt{3}}{2}\right)\right) = (1, \sqrt{3})$

since $B(1, \sqrt{3})$ and $D(-2, 0)$, the segment BD

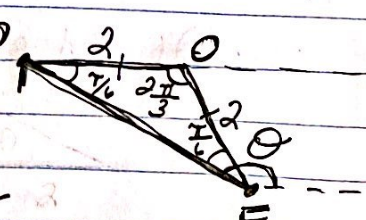
$\hookrightarrow BD = \sqrt{(1+2)^2 + (\sqrt{3}-0)^2} = \sqrt{9+3} = \sqrt{12} = \boxed{2\sqrt{3}}$

iv.) $\arg(z_2 - z_1)$ corresponds to $\overrightarrow{AB}$

$\therefore \theta = \frac{2\pi}{3}$

$\therefore \arg(z_2 - z_1) = \boxed{\frac{2\pi}{3}}$

v.) $\arg(z_4 - z_6)$ corresponds to $\overrightarrow{FD}$



isosceles Δ with $F\hat{O}D = \frac{2\pi}{3}$ $\therefore$ the other angles are $\frac{\pi}{6}$.

$\therefore \arg(z_4 - z_6) = \frac{2\pi}{3} + \frac{\pi}{6}$

$= \frac{4\pi}{6} + \frac{\pi}{6} = \boxed{\frac{5\pi}{6}}$