HW H.1 1abef, 4def, 5, 11 & H.2 1ghi, 2c

H.1

1.) a.) $4 \rightarrow |4| = 4$ & $\arg(4) = 0$ (on positive axis)

$\therefore \boxed{4 \text{ cis } 0}$

b.) $4i \rightarrow |4i| = 4$ & $\arg(4i) = \frac{\pi}{2}$ (on positive imaginary axis)

$\therefore \boxed{4 \text{ cis } \frac{\pi}{2}}$

e.) $1+i \rightarrow |1+i| = \sqrt{(1)^2+(1)^2} = \sqrt{2}$ & $\arg(1+i) = \frac{\pi}{4}$ (same x & y value)

$\therefore \boxed{\sqrt{2} \text{ cis } \frac{\pi}{4}}$

f.) $2-2i \rightarrow |2-2i| = \sqrt{(2)^2+(-2)^2} = \sqrt{8} = 2\sqrt{2}$ & $\arg(2-2i) = -\frac{\pi}{4}$ (same x, -y value)

$\therefore \boxed{2\sqrt{2} \text{ cis}\left(-\frac{\pi}{4}\right)}$

4.) d.) $3 \text{ cis } 0 = 3(\cos 0 + i \sin 0) = 3(1 + 0i) = \boxed{3}$

e.) $\sqrt{2} \text{ cis}\left(-\frac{\pi}{4}\right) = \sqrt{2}\left(\cos\left(-\frac{\pi}{4}\right) + i \sin\left(-\frac{\pi}{4}\right)\right) = \sqrt{2}\left(\frac{\sqrt{2}}{2} - \frac{\sqrt{2}}{2}i\right)$

$= \boxed{1 - i}$

f.) $\sqrt{3} \text{ cis } \frac{2\pi}{3} = \sqrt{3}\left(\cos \frac{2\pi}{3} + i \sin \frac{2\pi}{3}\right) = \sqrt{3}\left(-\frac{1}{2} + \frac{\sqrt{3}}{2}i\right)$

$= \boxed{-\frac{\sqrt{3}}{2} + \frac{3}{2}i}$

5.) a.) $(-2, 2\sqrt{3})$ $|-2+2\sqrt{3}i| = \sqrt{(-2)^2+(2\sqrt{3})^2} = \sqrt{4+12} = \sqrt{16} = 4$

$\arg(-2+2\sqrt{3}i) = \frac{2\pi}{3}$ $(-2, 2\sqrt{3})$ when divided by 4 is $\left(\frac{-1}{2}, \frac{\sqrt{3}}{2}\right)$

$\therefore \boxed{4 \operatorname{cis} \frac{2\pi}{3}}$

which is the point of $\frac{2\pi}{3}$

b.) i. $|z_2| = 2|z_1| = 2(4) = 8$

$\arg(z_2) = \arg(z_1) + \frac{\pi}{2} = \frac{2\pi}{3} + \frac{\pi}{2} = \frac{4\pi}{6} + \frac{3\pi}{6} = \frac{7\pi}{6} = \frac{-5\pi}{6}$

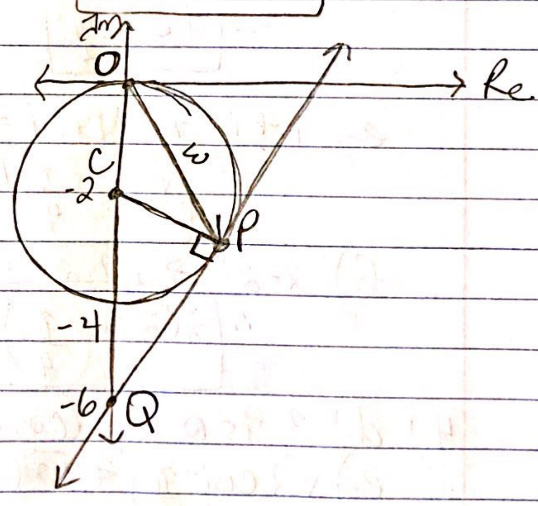
$\therefore \boxed{8 \operatorname{cis}\left(-\frac{5\pi}{6}\right)}$

(arg must be between π and $-\pi$)

ii. $8\operatorname{cis}\left(\frac{-5\pi}{6}\right) \Rightarrow 8\left(\cos\left(\frac{-5\pi}{6}\right) + i\sin\left(\frac{-5\pi}{6}\right)\right)$

$\Rightarrow 8\left(\frac{-\sqrt{3}}{2} - \frac{1}{2}i\right) \Rightarrow \boxed{-4\sqrt{3} - 4i}$

11.) a.) $\boxed{|z+2i| = 2}$



Explanation: $|z| = r$ gives the equation of a circle about the origin with radius r for all arg(z). Since this circle has a center at (0, -2) and the equation of a circle with center (a, b) is $(x-a)^2 + (y-b)^2 = r^2$, we convert -2 to 2 just like we switch the signs of x-intercepts when writing them as factors.

b.) i.) Connect the center of the circle to point P. If we label the point at (0, -6) Q, then we know CP = 2 and CQ = 4. $\therefore$ using pythagorean theorem $(2)^2 + (QP)^2 = (4)^2$

$\therefore QP = \sqrt{16-4} = \sqrt{12} = \boxed{2\sqrt{3}}$

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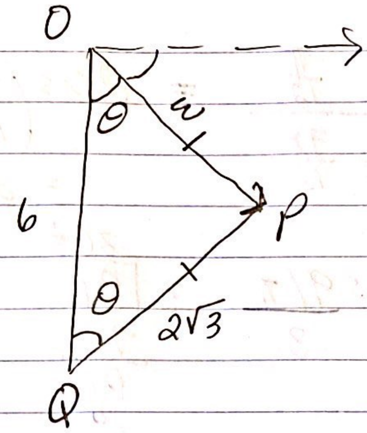
$\therefore \cos C\hat{Q}P = \frac{2\sqrt{3}}{4} = \frac{\sqrt{3}}{2}$

Now look at triangle OPQ



using cosine rule...

$$OP = \sqrt{(6)^2 + (2\sqrt{3})^2 - 2(6)(2\sqrt{3})\cos O\hat{Q}P}$$

$$= \sqrt{36 + 12 - 24\sqrt{3}\left(\frac{\sqrt{3}}{2}\right)} = \sqrt{48 - 36} = \sqrt{12} = \boxed{2\sqrt{3}}$$

ii.) $\arg(w)$ (Triangle OPQ is isosceles since $|OP| = |QP|$ and therefore $Q\hat{O}P = O\hat{Q}P$)

$\cos^{-1}\left(\frac{\sqrt{3}}{2}\right) = \frac{\pi}{6}$ since the cosine of $\frac{\pi}{6}$ is $\frac{\sqrt{3}}{2}$

$$\therefore \arg(w) = -\frac{\pi}{2} + \frac{\pi}{6} = \frac{-3\pi}{6} + \frac{\pi}{6} = \frac{-2\pi}{6} = \boxed{\frac{-\pi}{3}}$$

iii.) Hence write w in the form $a+bi$

$$2\sqrt{3}\,\text{cis}\left(\frac{-\pi}{3}\right) \Rightarrow 2\sqrt{3}\left(\cos\left(\frac{-\pi}{3}\right) + i\sin\left(\frac{-\pi}{3}\right)\right)$$

$$\Rightarrow 2\sqrt{3}\left(\frac{1}{2} - \frac{\sqrt{3}}{2}i\right) \Rightarrow \boxed{\sqrt{3} - 3i}$$

H.2

1.) g.) $\sqrt{3}\,\text{cis}\left(\frac{-\pi}{15}\right) \times 2\,\text{cis}\left(\frac{11\pi}{15}\right) = 2\sqrt{3}\,\text{cis}\left(\frac{-\pi}{15} + \frac{11\pi}{15}\right)$

$$= 2\sqrt{3}\,\text{cis}\left(\frac{10\pi}{15}\right) = 2\sqrt{3}\,\text{cis}\left(\frac{2\pi}{3}\right)$$

$$\Rightarrow 2\sqrt{3}\left(\cos\left(\frac{2\pi}{3}\right) + i\sin\left(\frac{2\pi}{3}\right)\right) \Rightarrow 2\sqrt{3}\left(\frac{-1}{2} + \frac{\sqrt{3}}{2}i\right) \Rightarrow \boxed{-\sqrt{3} + 3i}$$

h.) $\frac{\text{cis}\frac{\pi}{2}}{\text{cis}\frac{\pi}{3}} = \text{cis}\left(\frac{\pi}{2} - \frac{\pi}{3}\right) \Rightarrow \text{cis}\left(\frac{3\pi}{6} - \frac{2\pi}{6}\right) = \text{cis}\left(\frac{\pi}{6}\right)$

$$= \boxed{\frac{\sqrt{3}}{2} + \frac{1}{2}i}$$

i.) $\dfrac{4 \operatorname{cis} \frac{\pi}{12}}{2 \operatorname{cis} \frac{7\pi}{12}} = 2 \operatorname{cis}\left(\frac{\pi}{12} - \frac{7\pi}{12}\right) = 2 \operatorname{cis}\left(\frac{-6\pi}{12}\right) = 2 \operatorname{cis}\left(\frac{-\pi}{2}\right)$

$= 2\left(\cos\left(\frac{-\pi}{2}\right) + i\sin\left(\frac{-\pi}{2}\right)\right) = 2(0 - 1i) = \boxed{-2i}$

2c.) $\operatorname{cis} \frac{91\pi}{3}$

$$\begin{array}{r} 30\tfrac{1}{3} \\ 3 \overline{)\,91} \\ -9 \\ \hline 01 \\ -0 \\ \hline 1 \end{array}$$

$\therefore \operatorname{cis} 30\frac{1}{3}\pi = \operatorname{cis}\left(30\pi + \frac{\pi}{3}\right)$

$= \operatorname{cis}\left(\frac{\pi}{3}\right) = \boxed{\frac{1}{2} + \frac{\sqrt{3}}{2} i}$