HW 12F: F.2 # 5bd, 7, F.3 # 1.dcf, 2c, 6

5.) b.) $\underbrace{2\times 2 \quad 2\times 2}_{\checkmark} = 2\times 2$ $\begin{pmatrix} 2 & 1 \\ 0 & -1 \end{pmatrix}\begin{pmatrix} 1 & -3 \\ 2 & -1 \end{pmatrix} = \begin{bmatrix} 2+2 & -6-1 \\ 0-2 & 0+1 \end{bmatrix}$

$= \boxed{\begin{pmatrix} 4 & -7 \\ -2 & 1 \end{pmatrix}}$

d.) $\underbrace{3\times 3, \quad 3\times 1}_{\checkmark} = 3\times 1$ $\begin{pmatrix} 1 & 0 & -1 \\ -1 & 1 & 0 \\ 0 & -1 & 1 \end{pmatrix}\begin{pmatrix} 2 \\ 3 \\ 4 \end{pmatrix} = \begin{pmatrix} 2+0-4 \\ -2+3+0 \\ 0-3+4 \end{pmatrix}$

$= \boxed{\begin{pmatrix} -2 \\ 1 \\ 1 \end{pmatrix}}$

7.) a.) $P = \begin{pmatrix} 12.50 \\ 9.50 \end{pmatrix}$ $N = \begin{pmatrix} 2375 & 5156 \\ 2502 & 3612 \end{pmatrix}$

b.) $\begin{pmatrix} 2375 & 5156 \\ 2502 & 3612 \end{pmatrix}\begin{pmatrix} 12.50 \\ 9.50 \end{pmatrix} = \begin{bmatrix} 29687.5 + 48982 \\ 31275 + 34314 \end{bmatrix} = \boxed{\begin{pmatrix} 78669.5 \\ 65589 \end{pmatrix}}$

$\underbrace{2\times 2 \quad 2\times 1}$ $= 2\times 1$ — the total ticket sales Day 1 & Day 2 c.) $98669.5 + 65589 = \boxed{£144,258.50}$

1.) d.) $A(A^2 + A - 2I)$
$= A^3 + A^2 - 2AI$
$= \boxed{A^3 + A^2 - 2A}$

e.) $(A+B)(C+D) = \boxed{AC + AD + BC + BD}$

f.) $(A+B)^2 = (A+B)(A+B)$
$= \boxed{A^2 + AB + BA + B^2}$

2.) c.) If $C^2 = 4C - 3I$, find C^3 and C^5 in linear form.

$C^3 = C \times C^2 = C(4C - 3I)$
$= 4C^2 - 3C$
$= 4(4C - 3I) - 3C$
$= 16C - 12I - 3C$
$= \boxed{13C - 12I}$

$C^4 = C(13C - 12I)$
$= 13C^2 - 12C$
$= 13(4C - 3I) - 12C$
$= 52C - 39I - 12C$
$= 40C - 39I$

$\therefore C^5 = C(40C - 39I)$
$= 40C^2 - 39C$
$= 40(4C - 3I) - 39C$
$= 160C - 120I - 39C$
$= \boxed{121C - 120I}$

6.) a.) $A = \begin{pmatrix} 1 & 2 \\ -1 & 2 \end{pmatrix}$

$A^2 = \begin{pmatrix} 1 & 2 \\ -1 & 2 \end{pmatrix}\begin{pmatrix} 1 & 2 \\ -1 & 2 \end{pmatrix} = \begin{pmatrix} 1-2 & 2+4 \\ -1-2 & -2+4 \end{pmatrix} = \begin{pmatrix} -1 & 6 \\ -3 & 2 \end{pmatrix}$

$\therefore \begin{pmatrix} -1 & 6 \\ -3 & 2 \end{pmatrix} = a\begin{pmatrix} 1 & 2 \\ -1 & 2 \end{pmatrix} + b\begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$

$\begin{pmatrix} -1 & 6 \\ -3 & 2 \end{pmatrix} = \begin{pmatrix} a & 2a \\ -a & 2a \end{pmatrix} + \begin{pmatrix} b & 0 \\ 0 & b \end{pmatrix} \Rightarrow \begin{pmatrix} -1 & 6 \\ -3 & 2 \end{pmatrix} = \begin{pmatrix} a+b & 2a \\ -a & 2a+b \end{pmatrix}$

$\therefore a + b = -1$ & $2a = 6$
$3 + b = -1$
$\boxed{b = -4}$ $\boxed{a = 3}$

6.) b.) $A = \begin{pmatrix} 3 & 1 \\ 2 & -2 \end{pmatrix}$

$$A^2 = \begin{pmatrix} 3 & 1 \\ 2 & -2 \end{pmatrix}\begin{pmatrix} 3 & 1 \\ 2 & -2 \end{pmatrix} = \begin{pmatrix} 9+2 & 3-2 \\ 6-4 & 2+4 \end{pmatrix} = \begin{pmatrix} 11 & 1 \\ 2 & 6 \end{pmatrix}$$

$$\therefore \begin{pmatrix} 11 & 1 \\ 2 & 6 \end{pmatrix} = a\begin{pmatrix} 3 & 1 \\ 2 & -2 \end{pmatrix} + b\begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$$

$$\Rightarrow \begin{pmatrix} 11 & 1 \\ 2 & 6 \end{pmatrix} = \begin{pmatrix} 3a & a \\ 2a & -2a \end{pmatrix} + \begin{pmatrix} b & 0 \\ 0 & b \end{pmatrix}$$

$$\Rightarrow \begin{pmatrix} 11 & 1 \\ 2 & 6 \end{pmatrix} = \begin{pmatrix} 3a+b & a \\ 2a & -2a+b \end{pmatrix}$$

$\therefore \quad 3a+b = 11 \;\&\; \boxed{a=1}$

$3+b=11$

$\boxed{b=8}$