$h \to 0$

HW 17D: 3cd, 4, 5   17E:

3c.) $f(x) = 2x - x^2$ when $x = 1$

$f(1) = 2(1) - (1)^2 = 2 - 1 = 1 \quad \therefore \quad A = (1,1),\ B = (1+h, f(1+h))$

$$\lim_{h \to 0} \frac{f(a+h) - f(a)}{h} = \frac{f(1+h) - f(1)}{h} = \frac{2(1+h) - (1+h)^2 - 1}{h}$$

$$\Rightarrow \frac{2 + 2h - (1 + 2h + h^2) - 1}{h} \Rightarrow \frac{\cancel{2} + \cancel{2h} - \cancel{1} - \cancel{2h} - h^2 - \cancel{1}}{h}$$

$$\Rightarrow \frac{-h^2}{h} = -h \quad \therefore \quad \lim_{h \to 0} (-h) = -(0) = \boxed{0}$$

3d.) $f(x) = x^2 - 3x$ when $x = 0$ $\quad \therefore A = (0,0)$ $B = (h, f(h))$

$f(0) = (0)^2 - 3(0) = 0 - 0 = 0$

$$\lim_{h \to 0} \frac{f(h) - 0}{h} = \frac{(h)^2 - 3(h) - 0}{h} = \frac{h^2 - 3h}{h} = h - 3$$

$$\therefore \lim_{h \to 0} (h-3) = 0 - 3 = \boxed{-3}$$

4.) a.) $(1+h)^3 = (1+h)^2(1+h)$

$= (1 + 2h + h^2)(1+h) = 1 + 2h + h^2 + h + 2h^2 + h^3$

$= \boxed{h^3 + 3h^2 + 3h + 1}$

b.) $F(1,1)$, $M(1+h, f(1+h))$

i.) $FM = \dfrac{f(1+h) - 1}{1+h-1} = \dfrac{(1+h)^3 - 1}{h} = \dfrac{h^3 + 3h^2 + 3h + 1 - 1}{h}$

$= \boxed{\dfrac{h^3 + 3h^2 + 3h}{h}}$

ii. $\lim_{h \to 0} \dfrac{h^3 + 3h^2 + 3h}{h} \Rightarrow h^2 + 3h + 3$

$\therefore \lim_{h \to 0} (h^2 + 3h + 3) = 0 + 0 + 3 = \boxed{3}$

5.) $\dfrac{(x)\,1}{(x)\,x+h} - \dfrac{(x+h)\,1}{(x+h)\,x} = \dfrac{-h}{x(x+h)}$ (multiply by other denominator to convert to a common denominator)

$$\frac{x}{x(x+h)} - \frac{x+h}{x(x+h)} = \frac{x - x - h}{x(x+h)} = \boxed{\frac{-h}{x(x+h)}}$$ ← f(a+h)−f(a)

b.) i.) when $x = 1$ use the formula from (a.)

$\lim_{h \to 0} \dfrac{\frac{-h}{1(1+h)}}{h} = \dfrac{-\cancel{h}}{(1+h)} \cdot \dfrac{1}{\cancel{h}} = \dfrac{-1}{1+h} \quad \therefore \lim_{h \to 0} \Rightarrow \dfrac{-1}{1+0} = \dfrac{-1}{1}$

$= \boxed{-1}$

(ii.) $\lim_{h \to 0} \dfrac{\frac{-h}{3(3+h)}}{h} = \dfrac{-\cancel{h}}{3(3+h)} \cdot \dfrac{1}{\cancel{h}} = \dfrac{-1}{3(3+h)} \quad \therefore \lim_{h \to 0} \Rightarrow \dfrac{-1}{3(3)} = \boxed{\dfrac{-1}{9}}$

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17E HW: 3, 4

3.) a.) $f(3)$ = positive b.) negative (slightly sloping down)
c.) $f(-4)$ = negative d.) positive (curve is increasing at -2)

4.) $f'(x) = 2x+1$

a.) $f'(-2) = 2(-2)+1 = -4+1 = \boxed{-3}$ the slope of the tangent line when $x=-2$ is -3.

$f'(0) = 2(0)+1 = 0+1 = \boxed{1}$ the slope of the tangent line at $x=0$ is 1.

b.)