HW 18A: #3i, 4f, 5d, 6d, 8, 10, 13

3i.) $f(x) = \frac{x+5}{\sqrt{x}} \Rightarrow f(x) = \frac{x}{x^{\frac{1}{2}}} + \frac{5}{x^{\frac{1}{2}}} \Rightarrow f(x) = x^{\frac{1}{2}} + 5x^{-\frac{1}{2}}$

$\therefore f'(x) = \frac{1}{2}x^{-\frac{1}{2}} - \frac{5}{2}x^{-3/2} \Rightarrow f'(x) = \frac{1}{2\sqrt{x}} - \frac{5}{2x\sqrt{x}} \Rightarrow \boxed{f'(x) = \frac{x-5}{2x\sqrt{x}}}$

4f.) $y = (5-x)^2 \Rightarrow y = (5-x)(5-x) \Rightarrow y = 25 - 5x - 5x + x^2$

$\Rightarrow y = x^2 - 10x + 25 \therefore \boxed{\frac{dy}{dx} = 2x - 10}$

5d.) $\frac{dP}{du}$ for $P = \frac{5}{u} - 10u\sqrt{u} \Rightarrow P = 5u^{-1} - 10u^{\frac{3}{2}}$

$\therefore \frac{dP}{du} = -5u^{-2} - 15u^{\frac{1}{2}}$

$\Rightarrow \boxed{\frac{dP}{du} = \frac{-5}{u^2} - 15\sqrt{u}}$

6d.) $y = 2x^2 - 3x + 7$ at $x = -1$

$\frac{dy}{dx} = 4x - 3 \therefore 4(-1) - 3$

$= -4 - 3 = \boxed{-7}$

8.) $f(x) = -3x^2 + ax + b$ $(-3, 8)$ has gradient 9.

$f'(x) = -6x + a$

$\therefore 9 = -6(-3) + a$

$9 = 18 + a$

$\therefore \boxed{a = -9}$

$8 = -3(-3)^2 - 9(-3) + b$

$8 = -3(9) + 27 + b$

$8 = -27 + 27 + b$

$\therefore \boxed{b = 8}$

10.) $f(x) = ax + \frac{b}{x}$, $f(3) = 5$, $f'(1) = 5$

$f'(x) = a - \frac{b}{x^2}$ $\therefore$ $5 = 3a + \frac{b}{3}$ & $5 = a - b$

$\therefore a = 5 + b$ substitute

$5 = 3(5+b) + \frac{b}{3} \Rightarrow 5 = 15 + 3b + \frac{b}{3}$

$a = 5 - 3$

$\boxed{a = 2}$

$\Rightarrow -10 = \frac{10b}{3} \Rightarrow \frac{-30}{10} = \frac{10b}{10}$

$\boxed{b = -3}$

13.) $S = 2t^2 + 4t$

a.) $\boxed{\frac{dS}{dt} = 4t + 4}$; $\frac{dS}{dt}$ is the instantaneous rate of change in the car's position at time t

b.) $4(3) + 4$

$12 + 4 = \boxed{16}$; the instantaneous rate of change in the car's position is 16 m/s at time 3 s.