HW 18B.2: #3efg, 4bce, 5, 6

3e.) $y=6(5-x)^3$ $\quad y=6u^3 \quad u=5-x$

$\frac{dy}{du}=18u^2 \quad \frac{du}{dx}=-1 \quad \therefore \frac{dy}{dx}=-1(18u^2)=-18u^2$

$=\boxed{-18(5-x)^2}$

3f.) $y=\sqrt[3]{2x^3-x^2}$ $\quad y=\sqrt[3]{u} \quad u=2x^3-x^2$

$\therefore y=u^{1/3} \quad \frac{du}{dx}=6x^2-2x$

$\frac{dy}{du}=\frac{1}{3}u^{-2/3}$

$\therefore \frac{dy}{dx}=(6x^2-2x)\left(\frac{1}{3}u^{-2/3}\right) \Rightarrow \boxed{(6x^2-2x)\left(\frac{1}{3}(2x^3-x^2)^{-2/3}\right)}$

3g.) $y=\frac{6}{(5x-4)^2}$ $\quad y=\frac{6}{u^2} \quad u=5x-4$

$\frac{du}{dx}=5$

$\therefore y=6u^{-2}$

$\frac{dy}{du}=-12u^{-3} \quad \therefore \frac{dy}{dx}=5(-12u^{-3})$

$=-60u^{-3}$

$=\boxed{\frac{-60}{(5x-4)^3}}$

4b.) $y=(3x+2)^6$ $\quad y=u^6 \quad u=3x+2$

$\frac{dy}{du}=6u^5 \quad \frac{du}{dx}=3 \quad \therefore \frac{dy}{dx}=3(6u^5)=18(3x+2)^5$

at $x=-1 \Rightarrow 18(3(-1)+2)^5 \Rightarrow 18(-1)^5 \Rightarrow 18(-1)=\boxed{-18}$

4c.) $y=\frac{1}{(2x-1)^4}$ $\quad y=\frac{1}{u^4} \quad u=2x-1 \quad \therefore \frac{dy}{dx}=2(-4u^{-5})$

$\frac{du}{dx}=2$

$y=u^{-4} \qquad =-8(2x-1)^{-5}$

$\frac{dy}{du}=-4u^{-5}$

$=\frac{-8}{(2x-1)^5}$

at $x=1 \Rightarrow \frac{-8}{(2-1)^5}=\frac{-8}{(1)^5}=\boxed{-8}$

4e.) $y = \dfrac{4}{x+2\sqrt{x}}$ $\quad y = \dfrac{4}{u}$ $\quad u = x + 2\sqrt{x}$ $\quad \therefore \dfrac{dy}{dx} = \left(1+\dfrac{1}{\sqrt{x}}\right)\left(\dfrac{-4}{u^2}\right)$

$u = x + 2x^{\frac{1}{2}}$

$y = 4u^{-1}$ $\quad \dfrac{du}{dx} = 1 + x^{-\frac{1}{2}}$

$\dfrac{dy}{du} = \dfrac{-4}{u^2}$ $\quad = \left(1+\dfrac{1}{\sqrt{x}}\right)\left(\dfrac{-4}{(x+2\sqrt{x})^2}\right)$

at $x=4 \Rightarrow \left(1+\dfrac{1}{\sqrt{4}}\right)\left(\dfrac{-4}{(4+2\sqrt{4})^2}\right)$

$\Rightarrow \left(1+\dfrac{1}{2}\right)\left(\dfrac{-4}{(4+4)^2}\right) = \left(\dfrac{3}{2}\right)\left(\dfrac{-4}{64}\right) = \left(\dfrac{3}{2}\right)\left(\dfrac{-1}{16}\right) = \boxed{\dfrac{-3}{32}}$

5.) $f(x) = (2x-b)^a$ is $f'(x) = 24x^2 - 24x + 6$

$y = u^a$ $\quad u = 2x - b$

$\dfrac{dy}{du} = au^{a-1}$ $\quad \dfrac{du}{dx} = 2$ $\quad \therefore \dfrac{dy}{dx} = 2au^{a-1} \quad \therefore 2au^{a-1} = 6(4x^2 - 4x + 1)$

$= 6((4x^2 - 2x)(-2x+1))$

$= 6(2x(2x-1) - 1(2x-1))$

$\therefore 2au^{a-1} = 6(2x-1)^2$

$2a = 6$ or $a - 1 = 2$

$\therefore \boxed{a=3}$ $\quad \boxed{b=1}$ $\quad 2x - b = 2x - 1$

$-b = -1$

$b = 1$

6.) $y = \dfrac{a}{\sqrt{1+bx}}$ $\quad y = \dfrac{a}{\sqrt{u}}$ $\quad u = 1 + bx$ $\quad \therefore \dfrac{dy}{dx} = \dfrac{-ab}{2}u^{-3/2}$

$\dfrac{du}{dx} = b$

$y = au^{-\frac{1}{2}}$ $\quad \dfrac{dy}{dx} = \dfrac{-ab}{2(1+bx)^{3/2}}$

$\dfrac{dy}{du} = -\dfrac{a}{2}u^{-3/2}$

$\therefore -\dfrac{1}{8} = \dfrac{-ab}{2(1+3b)^{3/2}}$ $\quad \Rightarrow -2(1+3b)^{3/2} = -8ab$

$(1+3b)^{3/2} = 4ab$

plug in $(3,1) \rightarrow 1 = \dfrac{a}{\sqrt{1+3b}}$ $\quad \Rightarrow \sqrt{1+3b} = a$ Substitute

$\dfrac{(1+3b)^{3/2}}{\sqrt{1+3b}} = \dfrac{4b\sqrt{1+3b}}{\sqrt{1+3b}}$

$\Rightarrow 1 + 3b = 4b$ $\quad a = \sqrt{1+3}$

$\therefore \boxed{b=1}$ $\quad \boxed{a=2}$