18C HW: 1e, 2bf, 3b, 4, 6

1e.) $f(x) = x\sqrt{x^2-1}$

$u = x$ $\quad v = \sqrt{x^2-1}$ $\quad y = \sqrt{u}$ $\quad u = x^2-1$

$u' = 1$ $\quad v' = \dfrac{2x}{2\sqrt{x^2-1}}$ $\quad y' = \dfrac{1}{2\sqrt{u}}$ $\quad u' = 2x$

$v' \Rightarrow \dfrac{x}{\sqrt{x^2-1}}$

$\therefore \dfrac{dy}{dx} = (1)(\sqrt{x^2-1}) + (x)\left(\dfrac{x}{\sqrt{x^2-1}}\right)$

$\Rightarrow \sqrt{x^2-1} + \dfrac{x^2}{\sqrt{x^2-1}}$

$\Rightarrow \dfrac{\dfrac{x^2-1+x^2}{\sqrt{x^2-1}}}{\sqrt{x^2-1}} \Rightarrow \boxed{\dfrac{2x^2-1}{x^2-1}}$

2b.) $y = 4x(2x+1)^3$

$u = 4x$ $\quad v = (2x+1)^3$ $\quad y = u^3$ $\quad u = 2x+$

$u' = 4$ $\quad v' = 6(2x+1)^2$ $\quad y' = 3u^2$ $\quad u' = 2$

$\dfrac{dy}{dx} = (4)(2x+1)^3 + (4x)(6(2x+1)^2) \Rightarrow \boxed{4(2x+1)^3 + 24x(2x+1)^2}$

2f.) $y=\sqrt{x}(x-x^2)^3$ $u=\sqrt{x}$ $v=(x-x^2)^3$ $y=u^3$ $u=x-x^2$

$u'=\dfrac{1}{2\sqrt{x}}$ $v'=3(1-2x)(x-x^2)^2$ $y'=3u^2$ $u'=1-2x$

$$y'=\left(\frac{1}{2\sqrt{x}}\right)(x-x^2)^3+\sqrt{x}(3)(1-2x)(x-x^2)^2$$

$$\therefore \boxed{y'=\frac{(x-x^2)^3}{2\sqrt{x}}+3\sqrt{x}(1-2x)(x-x^2)^2}$$

3b.) $y=\sqrt{x}(x^2-x+1)^2$ at $x=4$ $u=\sqrt{x}$ $v=(x^2-x+1)^2$

$u'=\dfrac{1}{2\sqrt{x}}$ $y=u^2$ $u=x^2-x+1$

$y'=2u$ $u'=2x-1$

$$y'=\frac{1}{2\sqrt{x}}(x^2-x+1)^2+\sqrt{x}(2)(2x-1)(x^2-x+1) \therefore v'=2(2x-1)(x^2-x+1)$$

$$\Rightarrow \frac{1}{2\sqrt{4}}(4^2-4+1)^2+\sqrt{4}(2)(8-1)(16-4+1)$$

$$\Rightarrow \frac{1}{4}(13)^2+4(7)(13) \Rightarrow \frac{169}{4}+364 \Rightarrow 42\tfrac{1}{4}+364$$

$$=\boxed{406\tfrac{1}{4}}$$

4.) $y=\sqrt{x}(3-x)^2$ $u=\sqrt{x}$ $v=(3-x)^2$ $y=u^2$ $u=3-x$

$u'=\dfrac{1}{2\sqrt{x}}$ $v'=-2(3-x)$ $y'=2u$ $u'=-1$

$$y'=\frac{1}{2\sqrt{x}}(3-x)^2+\sqrt{x}(-2)(3-x) \Rightarrow \frac{(3-x)^2}{2\sqrt{x}}-\frac{2\sqrt{x}(3-x)\cdot 2\sqrt{x}}{2\sqrt{x}}$$

$$\Rightarrow \frac{(3-x)(3-x)-4x(3-x)}{2\sqrt{x}} \Rightarrow \frac{9-3x-3x+x^2-12x+4x^2}{2\sqrt{x}}$$

$$\Rightarrow \frac{5x^2-18x+9}{2\sqrt{x}}$$

45, −3, −15, −18

$(5x^2-3x)(-15x+9)$

$x(5x-3)-3(5x-3)$

$(x-3)(5x-3)$

$$\Rightarrow \boxed{\frac{(x-3)(5x-3)}{2\sqrt{x}}}$$

b.) the tangent is horizontal when the slope is 0.

$\therefore (x-3)(5x-3)=0$

$x-3=0$ $5x-3=0$

$\boxed{x=3}$ $\boxed{x=3/5}$

c.) D: $x>0$ (x cannot be negative bc the denominator will be undefined)

-original function D: $x\geq 0$.

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6.) $y=(x+3)(x-2)^2$ $\quad u=x+3 \quad v=(x-2)^2 \quad y=u^2 \quad u=x-2$

$u'=1 \quad v'=2(x-2) \quad y'=2u \quad u'=1$

$y'=1(x-2)^2+(x+3)(2x-4)$

$(x-2)^2+(x+3)(2x-4)=-7$

$\Rightarrow (x-2)(x-2)+2x^2-4x+6x-12=-7$

$x^2-4x+4+2x^2+2x-5=0$

$\Rightarrow 3x^2-2x-1=0$ $\quad -3 \quad 1 \quad -3 \quad -2$

$(3x^2-3x)(+x-1)=0$

$3x(x-1)+1(x-1)=0$

$(3x+1)(x-1)=0 \rightarrow \boxed{x=1,\ x=\frac{-1}{3}}$