HW 18D: 1be, 2d, 3bd, 6

1b.) $y = \frac{x^2}{2x+1}$ $\quad u = x^2 \quad v = 2x+1$

$u' = 2x \quad v' = 2$

$\therefore \frac{dy}{dx} = \frac{2x(2x+1) - 2(x^2)}{(2x+1)^2} \Rightarrow \frac{4x^2 + 2x - 2x^2}{(2x+1)^2} = \boxed{\frac{2x^2+2x}{(2x+1)^2}}$

1e.) $y = \frac{x^2-3}{3x-x^2}$ $\quad u = x^2-3 \quad v = 3x-x^2$

$u' = 2x \quad v' = 3-2x$

$\therefore \frac{dy}{dx} = \frac{2x(3x-x^2) - (3-2x)(x^2-3)}{(3x-x^2)^2} \Rightarrow \frac{6x^2 - 2x^3 - (3x^2 - 9 - 2x^3 + 6x)}{(3x-x^2)^2}$

$\Rightarrow \frac{6x^2 - 2x^3 - 3x^2 + 9 + 2x^3 - 6x}{(3x-x^2)^2} \Rightarrow \boxed{\frac{3x^2-6x+9}{(3x-x^2)^2}}$

2d.) $\frac{d}{dx}\left(\frac{4x}{\sqrt{x-5}}\right)$ $\quad u = 4x \quad v = \sqrt{x-5}$ (chain rule: $y = \sqrt{u}$, $u = x-5$)

$u' = 4 \quad v' = \frac{1}{2\sqrt{x-5}}$ $\quad y' = \frac{1}{2}u^{-\frac{1}{2}} \cdot u' = 1$

$\therefore \frac{d}{dx} = \frac{4\sqrt{x-5} - \frac{4x}{2\sqrt{x-5}}}{(\sqrt{x-5})^2} \Rightarrow \frac{\frac{4(x-5) - 2x}{\sqrt{x-5}}}{x-5} \Rightarrow \frac{4x-20-2x}{(x-5)^{3/2}}$

$\Rightarrow \boxed{\frac{2x-20}{(x-5)^{3/2}}}$

3b.) $y = \frac{x^3}{x^2+1}$ at $x=-1$ $\quad u = x^3$ $\quad v = x^2+1$

$u' = 3x^2$ $\quad v' = 2x$

$\therefore \frac{dy}{dx} = \frac{3x^2(x^2+1) - 2x(x^3)}{(x^2+1)^2}$ $\therefore \frac{3(-1)^2((-1)^2+1) - 2(-1)(-1)^3}{((-1)^2+1)^2}$

$\Rightarrow \frac{3(2)+2(-1)}{(2)^2} \Rightarrow \frac{6-2}{4} = \frac{4}{4} = \boxed{1}$

3d.) $y = \frac{x^2}{\sqrt{x^2+5}}$ at $x=-2$ $\quad u = x^2$ $\quad v = \sqrt{x^2+5}$ $\quad y = \sqrt{u}$ $\quad u = x^2+5$

$u' = 2x$ $\quad v' = \frac{2x}{2\sqrt{x^2+5}}$ $\quad y' = \frac{1}{2\sqrt{u}}$ $\quad u' = 2x$

$\therefore \frac{dy}{dx} = \frac{2x\sqrt{x^2+5} - x^2\left(\frac{x}{\sqrt{x^2+5}}\right)}{(\sqrt{x^2+5})^2}$ $\quad v' = \frac{x}{\sqrt{x^2+5}}$

$\Rightarrow \frac{\left[2x\sqrt{x^2+5} - \frac{x^3}{\sqrt{x^2+5}}\right]}{x^2+5} \Rightarrow \frac{\left[2(-2)\sqrt{(-2)^2+5} - \frac{(-2)^3}{\sqrt{(-2)^2+5}}\right]}{(-2)^2+5}$

$\Rightarrow \frac{-4(3)+\frac{8}{3}}{9} \Rightarrow \frac{-12+\frac{8}{3}}{9} \Rightarrow \frac{\frac{-36+8}{3}}{9} \Rightarrow -\frac{28}{3} \cdot \frac{1}{9} = \boxed{\frac{-28}{27}}$

6.) a.) $y = \frac{x^2+6}{2x+1}$ $\quad u = x^2+6$ $\quad v = 2x+1$ $\therefore \frac{dy}{dx} = \frac{2x(2x+1) - 2(x^2+6)}{(2x+1)^2}$

$u' = 2x$ $\quad v' = 2$

$\Rightarrow \frac{4x^2+2x-2x^2-12}{(2x+1)^2} \Rightarrow \boxed{\frac{2x^2+2x-12}{(2x+1)^2}}$

b.) i. zero

$2x^2+2x-12=0$

$2(x^2+x-6)=0$

$2(x+3)(x-2)=0$

$\boxed{x=-3}$ $\quad \boxed{x=2}$

both are in the domain b/c the denominator is not undefined at these values.

ii. undefined

$(2x+1)^2=0$

$2x+1=0$

$2x=-1$

$\boxed{x=\frac{-1}{2}}$