HW 18G: 2cegmn, 3ijkl, 4c, 5

2c.) $e^x \cos x$ $\quad u = e^x \quad v = \cos x$

$u' = e^x \quad v' = -\sin x$

$\boxed{e^x \cos x - e^x \sin x}$

2e.) $\ln(\sin x)$ $\quad y = \ln u \quad u = \sin x$

$y' = \frac{1}{u} \quad u' = \cos x$

$\boxed{\frac{\cos x}{\sin x}}$

2g.) $3 \tan \pi x$

$\Rightarrow 3 \cdot \pi \cdot \frac{1}{\cos^2 \pi x} = \boxed{\frac{3\pi}{\cos^2 \pi x}}$

2m.) $e^{\cos\sqrt{x}}$ $\quad y = e^u \quad u = \cos\sqrt{x}$

$y' = e^u \quad u' = \frac{-\sin\sqrt{x}}{2\sqrt{x}}$

$y' = -\sin u$

$y = \cos u$

$u = \sqrt{x}$

$u' = \frac{1}{2\sqrt{x}}$

$\therefore \boxed{\frac{-\sin\sqrt{x}\, e^{\cos\sqrt{x}}}{2\sqrt{x}}}$

2n.) $\frac{2\sqrt{x}}{\tan x}$ $\quad u = 2\sqrt{x} \quad v = \tan x$

$u' = \frac{1}{\sqrt{x}} \quad v' = \frac{1}{\cos^2 x}$

$\rightarrow \frac{\frac{\tan x}{\sqrt{x}} - \frac{2\sqrt{x}}{\cos^2 x}}{\tan^2 x} \frac{\cos x}{\cos x} \Rightarrow \frac{\frac{\sin x}{\sqrt{x}\cos x} - \frac{2\sqrt{x}}{\cos^2 x}}{\frac{\sin^2 x}{\cos^2 x}} \cdot \frac{\sqrt{x}}{\sqrt{x}} \Rightarrow$ next page

$\therefore$

2n. cont.) $\dfrac{\sin x\cos x - 2x}{\sqrt{x}\,\cancel{\cos^2 x}} \cdot \dfrac{\cancel{\cos^2 x}}{\sin^2 x}$ (multiply by reciprocal when dividing a fraction)

$\therefore \boxed{\dfrac{\sin x\cos x - 2x}{\sqrt{x}\,\sin^2 x}}$

3i.) $\cos(\cos x)$ $\quad y=\cos u \quad u=\cos x$ $\quad \therefore \boxed{\sin x \sin(\cos x)}$

$y'=-\sin u \quad u'=-\sin x$

3j.) $\cos^3 4x$ $\quad y=u^3 \quad u=\cos 4x$ $\quad \therefore \boxed{-12\sin 4x\cos^2 4x}$

$y'=3u^2 \quad u'=-4\sin 4x$

3k.) $\dfrac{2}{\sin^2 4x}$ $\quad y=\dfrac{2}{u^2} \quad u=\sin 4x$ $\quad \therefore \boxed{\dfrac{-16\cos 4x}{\sin^3 4x}}$

$u'=4\cos 4x$

$y=2u^{-2}$

$y'=\dfrac{-4}{u^3}$

3l.) $\dfrac{1}{3}\tan^2 2x$ $\quad y=\dfrac{1}{3}u^2 \quad u=\tan 2x$ $\quad \therefore \dfrac{4\tan 2x}{3\cos^2 2x} \Rightarrow \dfrac{4\sin 2x}{\cos 2x} \cdot \dfrac{1}{\cos^2 2x}$

$u'=\dfrac{2}{\cos^2 2x}$

$y'=\dfrac{2}{3}u$ $\quad \Rightarrow \boxed{\dfrac{4\sin 2x}{3\cos^3 2x}}$

4c.) $f(x)=\cos x\sin x$ $\quad u=\cos x \quad v=\sin x$

$u'=-\sin x \quad v'=\cos x$

$\therefore f'(x)=-\sin^2 x+\cos^2 x$ at $\dfrac{\pi}{4}$ $\Rightarrow -\left(\sin\dfrac{\pi}{4}\right)^2+\left(\cos\dfrac{\pi}{4}\right)^2$

$\Rightarrow -\left(\dfrac{\sqrt{2}}{2}\right)^2+\left(\dfrac{\sqrt{2}}{2}\right)^2 \Rightarrow -\dfrac{1}{2}+\dfrac{1}{2}=\boxed{0}$

5.) $f(x)=2\cos^2 x+2\sin^2 x+1$ $\quad y=2u^2 \quad u=\cos x$ & $y=2u^2 \quad u=\sin x$

$f'(x)=-4\cos x\sin x+4\cos x\sin x$ $\quad y'=4u \quad u'=-\sin x \quad y'=4u \quad u'=\cos x$

$=\boxed{0}$

$f(x)=2(\cos^2 x+\sin^2 x)+1$ $\quad \therefore$ if $f(x)=3$ $\quad$ (use the property that $\cos^2 x+\sin^2 x=1$)

$=2(1)+1$ $\quad f'(x)=0$ ✓

$=3$