HW 18H: 1cd, 2beh, 3, 4b, 8abc, 10

1c.) $f(x) = 2x^3 - 3x^2 - x + 5$

$f'(x) = 6x^2 - 6x - 1$

$\boxed{f''(x) = 12x - 6}$

1d.) $f(x) = \dfrac{2-3x}{x^2} = \dfrac{2}{x^2} - \dfrac{3}{x} = 2x^{-2} - 3x^{-1}$

$f'(x) = -4x^{-3} + 3x^{-2}$

$f''(x) = 12x^{-4} - 6x^{-3} \Rightarrow f''(x) = \dfrac{12}{x^4} - \dfrac{6}{x^3} \Rightarrow \boxed{f''(x) = \dfrac{12-6x}{x^4}}$

2b.) $y = x^2 - \dfrac{5}{x^2} = x^2 - 5x^{-2}$

$y' = 2x + 10x^{-3} \rightarrow y'' = 2 - 30x^{-4} \rightarrow \boxed{y'' = 2 - \dfrac{30}{x^4}}$

2e.) $y = (x^2 - 3x)^2$ easiest if you FOIL it out

$(x^2-3x)(x^2-3x) = x^4 - 3x^3 - 3x^3 + 9x^2 = x^4 - 6x^3 + 9x^2$

$\therefore y' = 4x^3 - 18x^2 + 18x$

& $\boxed{y'' = 12x^2 - 36x + 18}$

2h.) $y = \dfrac{1-e^{-x}}{x}$ $\quad u = 1 - e^{-x} \quad v = x$

$u' = e^{-x} \quad v' = 1$

$\therefore y' = \dfrac{xe^{-x} - (1-e^{-x})}{x^2} \Rightarrow \dfrac{xe^{-x} - 1 + e^{-x}}{x^2} \Rightarrow \dfrac{xe^{-x} + e^{-x} - 1}{x^2}$

$\therefore u = xe^{-x} + e^{-x} - 1 \quad v = x^2$

$\downarrow \quad v' = 2x$

$u = x \quad v = e^{-x} \quad \rightarrow \quad u' = \cancel{e^{-x}} - xe^{-x} - \cancel{e^{-x}}$

$u' = 1 \quad v' = -e^{-x} \quad u' = -xe^{-x}$

$e^{-x} - xe^{-x}$

$\therefore y'' = \dfrac{-x^3e^{-x} - 2x(xe^{-x} + e^{-x} - 1)}{x^4} \Rightarrow \dfrac{-x^3e^{-x} - 2x^2e^{-x} - 2xe^{-x} + 2x}{x^4}$

factor out an x $\Rightarrow \boxed{\dfrac{-x^2e^{-x} - 2xe^{-x} - 2e^{-x} + 2}{x^3}}$

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$\therefore u' = 12\cos x \sin x \quad v' = -\sin x$

$u = 6\sin^2 x \quad v = \cos x$

$\rightarrow 12\cos^2 x \sin x - 6\sin^3 x$

$\therefore \boxed{f''(x) = 12\cos^2 x \sin x - 6\sin^3 x + 3\sin x}$

3.) $f(x) = x^3 - 2x + 5$

a.) $f(2) = (2)^3 - 2(2) + 5 = 8 - 4 + 5 = \boxed{9}$

b.) $f'(x) = 3x^2 - 2$

$f'(2) = 3(2)^2 - 2 = 12 - 2 = \boxed{10}$

c.) $f''(x) = 6x$

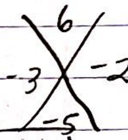
$f''(2) = 6(2) = \boxed{12}$

4b.) $f(x) = x^4 - 10x^3 + 36x^2 - 72x + 108$

$f'(x) = 4x^3 - 30x^2 + 72x - 72$

$f''(x) = 12x^2 - 60x + 72 \rightarrow 12(x^2 - 5x + 6) = 0$

$\rightarrow 12(x-3)(x-2) = 0 \quad \therefore \boxed{x = 2, 3}$



8a.) $y = 3\tan x \quad y' = \dfrac{3}{\cos^2 x} \quad y = 3u^{-2} \quad u = \cos x$

$y' = -6u^{-3} \quad u' = -\sin x$

$\therefore \boxed{y'' = \dfrac{6\sin x}{\cos^3 x}}$

8b.) $y = x\sin x \quad u = x \quad v = \sin x \quad \therefore y' = \sin x + x\cos x$

$u' = 1 \quad v' = \cos x$

$u = x \quad v = \cos x$

$u' = 1 \quad v' = -\sin x$

$\cos x - x\sin x$

$\therefore y'' = \cos x + \cos x - x\sin x$

$\boxed{y'' = 2\cos x - x\sin x}$

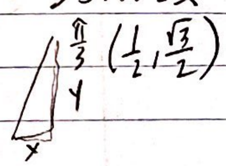
8c.) $y = \sin^2 3x \quad y = u^2 \quad u = \sin 3x \quad \therefore y' = 6\sin 3x \cos 3x$

$y' = 2u \quad u' = 3\cos 3x$

$u = 6\sin 3x \quad v = \cos 3x$

$u' = 18\cos 3x \quad v' = -3\sin 3x$

$\therefore y'' = 18\cos^2 3x - 18\sin^2 3x = \boxed{18(\cos^2 3x - \sin^2 3x)}$



10.) $f(x) = 2\sin^3 x - k\sin x \quad f'\left(\frac{\pi}{3}\right) = \frac{3}{4}$

$y = 2u^3 \quad u = \sin x$

$y' = 6u^2 \quad u' = \cos x$

$\therefore f'(x) = 6\sin^2 x \cos x - k\cos x \Rightarrow \frac{3}{4} = 6\left(\sin\frac{\pi}{3}\right)^2\left(\cos\frac{\pi}{3}\right) - k\left(\cos\frac{\pi}{3}\right)$

b.) $f'(x) = 6\sin^2 x \cos x - 3\cos x$

$u = 6\sin^2 x \quad v = \cos x$

$y = 6u^2 \quad u = \sin x \quad v' = -\sin x$

$y' = 12u \quad u' = \cos x$

up top

$\therefore \frac{3}{4} = 6\left(\frac{3}{4}\right)\left(\frac{1}{2}\right) - \frac{1}{2}k \rightarrow -\frac{3}{2} = -\frac{1}{2}k$

$\frac{3}{4} = \frac{18}{8} - \frac{1}{2}k \quad \therefore \boxed{k = 3}$

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