HW 12G: 2, 5, 6ijkl, 7de

2.) $\begin{pmatrix} 3 & -4 \\ 1 & 2 \end{pmatrix}\begin{pmatrix} 2 & 4 \\ -1 & 3 \end{pmatrix} = \begin{pmatrix} 6+4 & 12-12 \\ 2-2 & 4+6 \end{pmatrix} = \begin{pmatrix} 10 & 0 \\ 0 & 10 \end{pmatrix}$ $\therefore$ det $= 10$

$\frac{1}{10}\begin{pmatrix} 2 & 4 \\ -1 & 3 \end{pmatrix} = \boxed{\begin{pmatrix} \frac{1}{5} & \frac{2}{5} \\ \frac{-1}{10} & \frac{3}{10} \end{pmatrix}}$

5.) $A = \begin{pmatrix} 2 & 3 \\ 4 & 7 \end{pmatrix}$ a.) $|A| = 14-12 = \boxed{2}$

b.) $\frac{1}{2}\begin{pmatrix} 7 & -3 \\ -4 & 2 \end{pmatrix} = \boxed{\begin{pmatrix} 7/2 & -3/2 \\ -2 & 1 \end{pmatrix}}$

c.) $AA^{-1} = I \Rightarrow \begin{pmatrix} 2 & 3 \\ 4 & 7 \end{pmatrix}\begin{pmatrix} 7/2 & -3/2 \\ -2 & 1 \end{pmatrix} = \begin{pmatrix} 7-6 & -3+3 \\ 14-14 & -6+7 \end{pmatrix} = \boxed{\begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}}$ ✓

$A^{-1}A = I \Rightarrow \begin{pmatrix} 7/2 & -3/2 \\ -2 & 1 \end{pmatrix}\begin{pmatrix} 2 & 3 \\ 4 & 7 \end{pmatrix} = \begin{pmatrix} 7-6 & 21/2 - 21/2 \\ -4+4 & -6+7 \end{pmatrix} = \boxed{\begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}}$ ✓

6i.) $\begin{pmatrix} -1 & -1 \\ 2 & 3 \end{pmatrix}$ $\det = -3+2 = -1 \therefore -1\begin{pmatrix} 3 & 1 \\ -2 & -1 \end{pmatrix} = \boxed{\begin{pmatrix} -3 & -1 \\ 2 & 1 \end{pmatrix}}$

6j.) $\begin{pmatrix} 4 & 10 \\ 2 & 5 \end{pmatrix}$ $\det = 20-20 = 0 \therefore$ $\boxed{\text{the inverse does not exist}}$

6k.) $\begin{pmatrix} \frac{1}{2} & 0 \\ -\frac{1}{2} & \frac{1}{2} \end{pmatrix}$ $\det = \frac{1}{4} - 0 = \frac{1}{4} \therefore 4\begin{pmatrix} \frac{1}{2} & 0 \\ \frac{1}{2} & \frac{1}{2} \end{pmatrix} = \boxed{\begin{pmatrix} 2 & 0 \\ 2 & 2 \end{pmatrix}}$

6l.) $\begin{pmatrix} 0 & \frac{1}{2} \\ -2 & 0 \end{pmatrix}$ $\det = 0 - 1 = -1 \therefore -1\begin{pmatrix} 0 & -1/2 \\ 2 & 0 \end{pmatrix} = \boxed{\begin{pmatrix} 0 & 1/2 \\ -2 & 0 \end{pmatrix}}$

involutory matrix :)

7d.) $A = \begin{pmatrix} k-2 & k \\ -3 & k \end{pmatrix}$ $\det = k(k-2) + 3k$

$k^2 - 2k + 3k$

$= k^2 + k$

$k^2 + k \neq 0$

$\therefore k(k+1) \neq 0$

$\therefore \boxed{k \neq 0 \text{ \& } k \neq -1}$

$A^{-1} = \frac{1}{k^2+k}\begin{pmatrix} k & -k \\ 3 & k-2 \end{pmatrix} = \boxed{\begin{pmatrix} \frac{k}{k^2+k} & \frac{-k}{k^2+k} \\ \frac{3}{k^2+k} & \frac{k-2}{k^2+k} \end{pmatrix}}$

7c.) $A = \begin{pmatrix} k^2 & k-1 \\ 2k & 1 \end{pmatrix}$

$\det A = k^2 - 2k(k-1)$

$= k^2 - 2k^2 + 2k$

$= -k^2 + 2k$

$\therefore -k^2 + 2k \neq 0$

$\therefore k(-k+2) \neq 0$

$\therefore \boxed{k \neq 0 \ \& \ k \neq 2}$

$A^{-1} = \dfrac{1}{-k^2+2k} \begin{pmatrix} 1 & -(k-1) \\ -2k & k^2 \end{pmatrix} = \begin{pmatrix} \dfrac{-1}{k^2-2k} & \dfrac{k-1}{k^2-2k} \\ \dfrac{2k}{k^2-2k} & \dfrac{-k^2}{k^2-2k} \end{pmatrix}$

$\Rightarrow \dfrac{-1}{k^2-2k}$