HW 6J : 1, 4, 8, 12

1.) $H(t) = -4t^2 + 4t + 3 \ , \ t \geq 0$

a.) $\boxed{\text{3 metres}}$ (the y-intercept represents the starting height)

b.) $\frac{-b}{2a} = \frac{-(4)}{2(-4)} = \frac{-4}{-8} = \frac{1}{2}$ $\therefore$ $\boxed{\text{0.5 seconds}}$

c.) $-4\left(\frac{1}{2}\right)^2 + 4\left(\frac{1}{2}\right) + 3 \Rightarrow -4\left(\frac{1}{4}\right) + 2 + 3 = -1 + 5 = \boxed{\text{4 metres}}$

d.) $0 = -4t^2 + 4t + 3 \Rightarrow (-4t^2 + 6t)(-2t + 3) = 0$

$-2t(2t-3) - 1(2t-3) = 0$

$(-2t-1)(2t-3) = 0$

$t = -\frac{1}{2}$ $\quad$ $t = 3/2$

-12

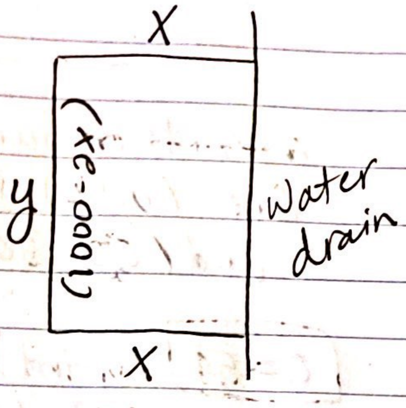
6 $\quad$ -2

4

only the positive solution makes sense.

$\boxed{\text{1.5 seconds}}$

4.)



$1000 = 2x + y \quad \therefore \quad y = 1000 - 2x$

$A = x(1000 - 2x)$
$A = 1000x - 2x^2$
$A = -2x^2 + 1000x$

$\frac{-b}{2a} = \frac{-1000}{2(-2)} = \frac{-1000}{-4} = 250$

$\therefore \quad y = 1000 - 2(250)$
$y = 1000 - 500$
$y = 500$

The dimensions that maximise area are $\boxed{250m \times 500m}$

8.) $y = ax^2 + bx + c$ x-intercepts at $-3, 3$ y-intercept at 8

a.) i. $c = 8$ (c is the y-intercept)

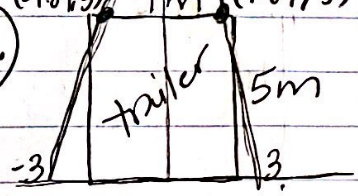
$\therefore y = a(x-3)(x+3) \Rightarrow 8 = a(-3)(3)$ (plug in 0 for x)
$\Rightarrow 8 = -9a$
$\frac{-8}{9} = a$

$\therefore y = \frac{-8}{9}(x-3)(x+3)$
$= \left(\frac{-8}{9}x + \frac{8}{3}\right)(x+3)$
$y \Rightarrow \frac{-8}{9}x^2 - \frac{8}{3}x + \frac{8}{3}x + 8 \Rightarrow \boxed{y = \frac{-8}{9}x^2 + 8}$

Also think vertex $(0,8)$ means $y = a(x-h)^2 + k$ translates to $y = ax^2 + 8$ since (h,k) is the vertex

ii. $b = 0$ iii. $a = \frac{-8}{9}$

b.)

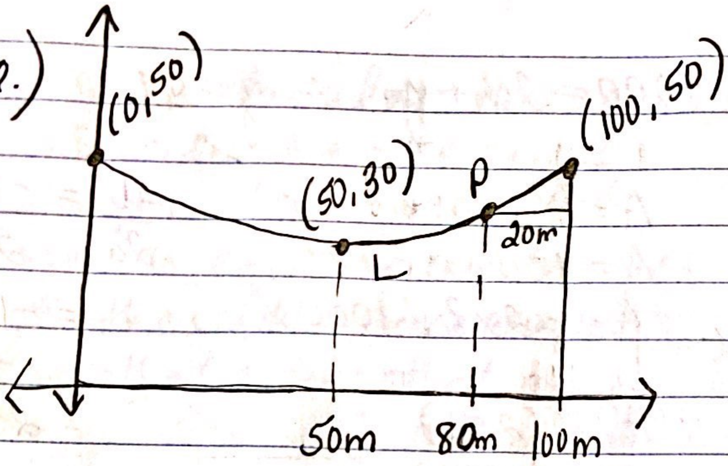


We need to test if the trailer will fit, which means the distance between the ends of the tunnel needs to be >4m at a height of 5m, which means the x-values need to be $> \pm 2$.

$5 = \frac{-8}{9}x^2 + 8$
$-8 \qquad -8$
$-3 = -\frac{8}{9}x^2$
$\frac{27}{8} = x^2$
$x = \pm\sqrt{\frac{27}{8}} \approx 1.84$

$\therefore$ No, the opening at 5m high will only be about 3.67m wide.

12.)



∴ Points on our graph are (0,50), (100,50) and (50,30)

$\boxed{c=50}$ (y-int)

∴ $50 = a(100)^2 + b(100) + 50$

& $30 = a(50)^2 + b(50) + 50$

$\Rightarrow 10{,}000a + 100b + 50 = 50$

$2500a + 50b + 50 = 30$

$\Rightarrow 10{,}000a + 100b = 0$

$2500a + 50b = -20 \; (\cdot 2)$

$\Rightarrow 10{,}000a + 100b = 0$

$+ \; -5{,}000a - 100b = 40$

$5{,}000a = 40$

$\therefore a = \frac{40}{5{,}000} = \frac{1}{125}$

$\therefore 2500\left(\frac{1}{125}\right) + 50b + 50 = 30$

$\Rightarrow 20 + 50b + 50 = 30$

$\Rightarrow 50b + 70 = 30$

$\Rightarrow 50b = -40$

$\therefore \boxed{b = -\frac{4}{5}}$

a.) $\boxed{y = \frac{1}{125}x^2 - \frac{4}{5}x + 50}$

b.) $y = \frac{1}{125}(80)^2 - \frac{4}{5}(80) + 50$

$= 51.2 - 64 + 50$

$= \boxed{37.2m}$

c.) $\boxed{\{x \mid 0 \le x \le 100\}}$

(the distance between the platforms)